FERTILITY MODELS WITH SOCIAL PARAMETERS

James M. Beshers, M.I.T.

I

In this paper we represent three aspects of fertility behavior as functions of social variables: first, the diffusion of family planning across a society; second, the determination of an initial ideal family size for a planning couple; and third, the achievement of a given family size throughout the child bearing cycle of a planning couple.

The first formulation is drawn from an earlier paper;[1] it is empirically suggested by research by Keyfitz.[2] The second formulation represents fairly conventional theories of the transmission of values in sociology; much American survey data on "ideal family size" provides the empirical substance.[3] The third formulation is an effort to represent the argument of J. A. Banks as presented in Prosperity and Parenthood.[4]

The social parameters will be defined over an n-class social system.[5] The set of classes C_1, C_2, C_n will be arranged from high to low with order determined by the matrix of probabilities of inter-marriage among classes.[6] An unambiguous order will be assumed to exist for this paper.[7] Further, it will be assumed that the distribution of other relevant contacts among classes will be approximately indexed by the distribution of marriages.[8]

II

Thus we shall define a concept of social distance over the inter-marriage matrix. Up to a rank order social distance will be defined by the subscripts of the classes. We shall further assume that a metric can be defined over the classes in two vectors, one representing upward distance and one representing downward distance. The elements of both vectors will be represented as d_{ij} with the relative magnitude of i and j determining the direction of the distance and $d_{ij} \geq 1$ if $i \neq j$. (The technical issues involved in the definition and estimation of such vectors are discussed elsewhere.)[9]

A similar analysis over a system of areas would utilize the volume of interchanges of migrants as indices of the "social distance" among inhabitants. Thus a village that has a high rate of interchange with a city will be "closer" to the city than a village with a low rate of interchange. Ideally we would like to combine social classes and areas in a single set of equations. It is just such a comparison that is made by Keyfitz.

III

Suppose family planning enters the social system in a given class, then spreads. For simplicity let us suppose that family planning enters at one "end" of the system, say C_1. Let us define the proportion planning in C_1 as ρ_1, in C_2 as ρ_2, and so on to C_n as ρ_n. Then at time t we have $\rho_1(t) > 0$, $\rho_2(t) = \rho_3(t) = - - - = \rho_n(t) = 0$.

Given that the order of classes is in the order defined by the "social distance" among them, then we expect that the "barriers" between classes will be broken down in the order of the subscripts.

In order to further specify the values of $\rho_i(t)$, we must supply within class diffusion parameters, we must supply a functional form for the rate of diffusion and we must specify a parameter that indexes the magnitude of the barrier.

One such formulation that has the merit of simplicity is the following:

If $\left(t - \sum_{i=2}^{n} \rho_{i-1,\, i}\right) > 0$

Then $\rho_i\left(t - \sum_{i=2}^{n} \rho_{i-1,\, i}\right) =$

$$1 - e^{-k_i\left(t - \sum_{i=2}^{n} \rho_{i-1,\, i}\right)}$$

If $\left(t - \sum_{i=2}^{n} \rho_{i-1,\, i}\right) < 0$

Then $\rho_i = 0$

where k_i is a parameter indexing the within class rate of diffusion and ρ is a parameter indexing the barrier between adjacent classes. We can define ρ as follows: $\rho_{i-1,\, i} =$

$$\frac{d_{i-1,\, i} + d_{i,\, i-1}}{2}$$

IV

For the proportion of a class that plans we shall define an ideal family size ζ_j in the class C_j. How shall we determine $\zeta_j(t)$?

Let us assume that $\zeta_j(t)$ is initially defined at the marriage of a couple and that we wish to date the marriage by the year of birth of the woman (the cohort of birth in the data of P. K. Whelpton). We shall use T as the time index for year of birth.

Thus we have ζ_j^T for each married couple. We shall allow this number to vary in successive cohorts, but it shall be a constant for the life cycle of any particular couple, independent of t.

We shall represent f^T as dependent upon the ascribed status of the couple, the achieved status of the couple, and prestige effects specific to the class membership of the couple.

To summarize: ideal family size for a given cohort of married women is determined by the successive application of three functions: f_1, f_2 and f_3.

The function f_1 specifies the influence of the values of the parents of the couple upon the values of the couple, the function f_2 specifies the influence of social mobility of the couple upon their ideal family size values, and the function f_3 specifies a prestige drift throughout the social system that changes ideal family size in the direction of the highest social classes.

Thus we have, aside from linear constants,

$$f_1: f_k^T = \frac{f_i^{T-G_i} + f_j^{T-G_j}}{2}$$

where C_k is the class expected for the couple conditional that the wife's parents are in Ci and the husband's parents are in Cj, where $f_i^{T-G_i}$ is the ideal family size of parents of wife and $f_j^{T-G_j}$ is the ideal family size of parents of husband, G stands for generation. Next,

$$f_2: f_h^T = \tfrac{1}{2} \left(f_k^t + \frac{f_h^{T-1}}{d_{k,h}} \right),$$

where f_h^{T-1} is last year's cohort ideal family size and $d_{k,h}$ is distance from C_k implied by parental status to C_h achieved by couple.

$$f_3: f_g^T = f_g^{T-1} + \frac{f_g^{T-1} - f_{g-1}^{T-1}}{d_{g-1,\,g}} +$$

$$\frac{f_g^{T-1} - f_{g-2}^{T-1}}{d_{g-2,g}} + \ldots\ldots\ldots$$

$$f_m^T = \frac{f_h^T + f_g^T}{2}$$

where f_g^{T-1} is last year's cohort value for the class achieved by the couple and the succeeding terms represent the influence of higher classes attenuated by a distance divisor. Thus f_m^T depends upon i, j, k, h and g.

If we include linear constants as weights for the influence of the three types of effect, then we have

$$f_1: f_k^T = a_1 f_i^{T-G_i} + (1-a_1) f_j^{T-G_j}$$

$$f_2: f_h^T = a_2 f_k^T + (1-a_2) \frac{f_h^{T-1}}{d_{k,h}}$$

$$f_3: f_g^T = f_g^{T-1} + \frac{f_g^{T-1} - f_{g-1}^{T-1}}{dg\text{-}1,\ g} +$$

$$\frac{f_g^{T-1} - f_{g-2}^{T-1}}{dg\text{-}2,g} + \ldots$$

$$f_m^T = a_3 f_h^T + (1-a_3) f_g^T$$

where $(0 \leq a_1 \leq 1)$
$(0 \leq a_2 \leq 1)$
$(0 \leq a_3 \leq 1)$

By suitably choosing the values of the constants we may express alternative theories for the influences of parents, present class, and prestige drift or fashion.

Consider the vector (a_1, a_2, a_3), in which we allow the elements to be either 0 or 1. Clearly a_2 and a_3 differentiate the theories alone, a_1 provides alternative theories for the relative influence of parents:

If $a_2 = 0$, then no parental influence;
" " $= 1$, then no mobility influence;
" $a_3 = 0$, then no parental or mobility influence; and
" " $= 1$, then no prestige influence.

In general we will allow the elements to range from 0 to 1 inclusive. We now have f_m^T defined - the cohort X class ideal family size.

Next let us consider the achievement of the ideal family size throughout the life of the cohort of planners.

In each year we have $p_{i,\,i+1}^{X}$, the probability of an additional child in a year at age x to a woman with i previous births. We shall define $p_{i\ i=1}^{x,\ldots} = 0$ if the woman is never married. At first marriage $p^x_{i,i+1} > 0$. We wish to make $p^x_{i\ i+1}$ depend upon the ideal family size τ, the expected future income I(x) and biological capacities declining with age h(x). One way to do this is to table h(x) for non-planning populations as a complete set of $p_{1,i+1}$. Then the effect of τ and of I(x) is to reduce a subset of values of $p^x_{i,i+1}$ to zero.

Let us first express p^x_i , i+1 as a function of τ and i. Then we will consider I(x), and then h(x).

$$g_1:\ p^x_{i,\ i+1} \begin{cases} > 0 \text{ if } \tau - i > 0 \\ = 0 \text{ if } \tau - i \leq 0 \end{cases}$$ Note i is a monotonically increasing function of x.

In order to specify g_2 we must introduce new definitions. For each cohort X class combination we have two constants expressed in dollars, S_j $\equiv$ standard of living for self and M_j $\equiv$ standard of living for a child. These two constants may be scaled to the annual income or to the total career income of the household. We further have a variable $I_j(x)$ $\equiv$ expected future income which may be scaled either to annual or total income. We should define a total career income curve and integrate for annual income, then do the same for S and M. In this way intermittent spacing to meet costs of college education can be represented in the model. Further, since we allow I(x) to be reestimated annually the fluctuations can be represented in the model.

It seems desirable that, at marriage, the values of I, S, M, and τ are compatible. This will be achieved over the career if, for any household,

$$I - S \geq M\tau$$

In order to preserve this relationship in the annual values we define

$$g_2:\ p^x_{i,i+1} > 0 \text{ if } (I_j(x)-S_j) \geq iM_j$$

$$p^x_{i\ i+1} = 0 \text{ if } (I-S) < iM.$$

Strictly speaking, Banks argues that S can be hedged to preserve M in the face of a drop in I(x) but this is incidental to our problem. Note that the difference between self-employed and salaried can be introduced in estimates of I(x).

The function g_3 comes from the analysis of non-planners

$$g_3:\ p^x_{i,i+1} = h\ (x)$$

where h (x) is a function representing the biological constraints of age and spacing upon birth probabilities assumed fixed for all cohorts. Thus,

$$p^x_{i,i+1} = 0 \text{ if } g_1 \text{ or } g_2 \text{ imply zero,}$$
$$= h(x) \text{ otherwise.}$$

Therefore we use h(x) as g_3 to introduce biological constraints into our birth projections for the planners, and we also use h(x) to express the births of the non-planners directly.

There are a number of sticky issues in the estimation of the parameters defined in this paper. However, in most cases the parameters may be directly estimated and indirectly estimated. For example, the proportion planning and the ideal family size may be directly estimated by appropriate surveys. These parameters may be indirectly estimated by the use of the model itself. With minor adjustments the model will deduce a whole series of demographic parameters, such as birth rates, average completed family sizes and child spacing that may be compared with such measures in a population. The implied values of some parameters are then indirectly estimated under the hypothesis that the other parameters are correctly estimated by calculating the values of the parameters in question that would be consistent with the demographic data; the degree of precision of such indirect estimation will, of course, depend upon the number of degrees of freedom that are available in turn the number of degrees of freedom are in part dependent upon one's confidence in his assumptions and direct estimates of other parameters.

Let me consider the estimation of two kinds of parameters in greater detail. The expected future income I(x) is a subjective phenomenon. Good direct estimates can only be made in carefully designed panel studies that utilize detailed theories of cognitive processes (the Bayesian concept of subjective probabilities is relevant here - we wish to predict the distribution of such numbers in a population). Ultimately we should make I(x) depend upon such numbers as income measured objectively in the population. However, if we recall that this parameter has two purposes: (1) to enable us to represent planned spacing; and (2) to enable us to represent hedging against down turns in expected future income, then we can test the model against data that we believe strongly reflect these two phenomena and attempt indirect estimation.

The biological function h(x) bears some relation to a number of computations that have been made by demographers on "non-contraceptive" populations. Ideally we would like to have cohort birth probabilities of the type that can be reconstructed from Whelpton's tables for the U.S. We would need these values for several different non-contraceptive populations so that we could take account of the effects of customs such as sexual abstinence or the sporadic use of birth control.

An alternative procedure might involve the use of a continuous stochastic process model, such as a semi-Markov process, fitted to demographic parameters of a non-contraceptive population. A discrete table of probabilities might then be calculated by integration.

This paper clearly leaves much unfinished business. Yet I hope that its main goal is accomplished, namely that some of you are now convinced that social and psychological parameters can be mathematically defined and introduced into birth projection models.

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Footnotes

1. "Birth Projections with Cohort Models" by James M. Beshers, paper read at the annual meeting of the Population Association of America, July 1964, forthcoming in Demography.
2. "A Factorial Arrangement of Comparisons of Family Size," by Nathan Keyfitz, American Journal of Sociology, Vol. 58 (March, 1953), pp. 470-479.
3. In particular we have the influence of parental status as expressed by Goldberg, upward mobility as expressed by Westoff, and a prestige drift that represents fashion. See D. Goldberg, "The Fertility of Two-Generation Urbanites," Population Studies, 12 (1959): 214-44, and Westoff, Charles, "The Changing Focus of Differential Fertility Research: The Social Mobility Hypothesis" in J. J. Spengler and O. D. Duncan (eds.) Population Theory and Policy (Glencoe, Ill.: Free Press, 1956).
4. Banks argues that expected future income, standard of living desired for self, and standard of living desired for children all must be taken into account to determine the influence of income upon patterns of child spacing as well as the influence upon total number of children. See J. A. Banks, Prosperity and Parenthood. (Humanities Press, N. Y., 1954).
5. See James M. Beshers and Stanley Reiter "Social Status and Social Change" Behavioral Science, Vol. 8, No. 1 (Jan. 1963).
6. Note that race, religion, nationality, occupation and other social variables may enter into the definition of these classes.
7. James M. Beshers "Urban Social Structure as a Single Hierarchy," Social Forces, Vol. 41, No. 3 (March, 1963) and Urban Social Structure (New York: Free Press, 1962).
8. Ibid.
9. Paper in progress in collaboration with Edward Laumann.